Two planes on merging routes are:
--different distances from the intersection
--traveling at the same speed.

SMART SKIES

Airspace Systems—Predicting Air Traffic Conflicts

Teacher Guide

Curriculum Supplement 2

Overview of Curriculum Supplement 2

You may choose to spread the experiment and calculation activities over two or three class periods, allowing time for setting up the experiment, conducting the experiment, doing the calculations, and discussing the outcomes.

This is the second in a series of Airspace Systems Curriculum Supplements that address distance-rate-and time problems. Each Curriculum Supplement consists of an experiment, worksheets to support the experiment, worksheets for paper-and-pencil calculations, a student analysis of the airspace scenario, and optional pre- and post-assessment instruments.

In this Curriculum Supplement, the controller must merge two flows of air traffic into one stream. The planes are each a different distance from the intersection and both are traveling at the same constant (fixed) speed.

Of the eight *Airspace Systems* Curriculum Supplements, this scenario is one step removed from the simplest case (Curriculum Supplement 1) in which the distances, as well as the speeds, are the same.

Airspace Scenario

Students will determine if two airplanes traveling on different merging routes will conflict with (meet) one another at the intersection of their flight routes.

The airplanes are each a **different distance from the point of intersection**.
The airplanes are traveling at the **same constant (fixed) speeds**.

Objectives

Students will determine the following:

> If two planes are traveling at the same constant (fixed) speed on two different routes and the planes are different distances from the point where the two routes come together, the planes will arrive at the intersection at different times. So the planes will not meet at the point where the routes come together.
>
> Also, since the planes are traveling at the same constant

(fixed) speed, their separation will remain the same. So at the intersection, the separation between the planes will be the same as their initial separation.

Introducing Your Students to the Airspace Scenario

If you have not already done so, you may want to show the "Gate to Gate" CD-ROM to introduce your students to the air traffic control system. (For more detail, see the Smart Skies Airspace Systems Introduction for Teachers.)

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, plan to meet at the movies. Ana lives 20 blocks from the theater. Alex lives 16 blocks from the theater. Ana and Alex will each leave their homes at the same time and walk at the same constant (fixed) speed.

You can ask your students these questions:

Will Ana and Alex arrive at the movie theater at the same time? Why or why not?
In particular, if your students think Ana and Alex will arrive at the same time, ask them to explain their reasoning.

Activity 2.0 --

Problem Statement

In a real-world scenario, each plane speed might be 400 nautical miles per hour. One plane might be 40 nautical miles from the point of intersection. The other plane might be 36 nautical miles from that point.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a "knot".

As a problem extension, you may want to ask your students to solve the problem using real-world data.

Problem Statement
Worksheet 2.0 describes and illustrates the airplane scenario. The speed of each airplane is 1/2 foot/second. One airplane is 20 feet from the point of intersection. The other airplane is 16 feet from the point of intersection.

Note: These speeds and distances were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.

Four questions are posed:

Q1: How many seconds will it take Flight WAL27 to travel 20 feet to the point where the routes come together?

Q2: How many seconds will it take Flight NAL63 to travel 16 feet to the point where the routes come together?

Q3: Will the planes meet at the point where the routes come together?

Q4: If not, how far apart will the planes be when the first plane reaches the point where the routes come together?

Student Handout:
Worksheet 2.0

Since the planes are traveling at the same constant (fixed) speed and each must travel a different distance to the point of intersection, students may be able to answer Question 3 correctly without answering Questions 1 and 2. That is, they may realize that the planes will not meet.

Materials
Worksheet 2.0: Problem Statement

Activity 2.1 -- Pretest

Estimated time:
15 - 30 minutes

The pretest is optional.

Instead of distributing the pretest, you may want to use the questions to guide a classroom discussion.

Student Handouts:
Worksheet 2.1A
Worksheet 2.1B

Pretest—Make a Prediction
The pretest steps the student through a careful reading of the airplane problem statement. The student is then asked to predict the outcome of the given airplane scenario.

The pretest may be assigned as either an individual or a small-group activity.

If your students have completed other Airspace Systems Curriculum Supplements, you may want to direct them to use a particular calculation method or methods to answer the pretest questions. In that case, Worksheet 2.1B contains blank vertical line plots as well as grids that students can use as they do their calculations.

Materials
Worksheet 2.1A: Pretest—Make A Prediction
Worksheet 2.1B: Lines and Grids

Activity 2.2 -- Experimentation

Estimated time:
Setup—30 minutes
Experiment—30 minutes

*For a step-by-step student orientation to the Experiment, see Curriculum Supplement 0, the introduction to **Airspace Systems**.*

Student Handouts:
Worksheet 2.2A
Worksheet 2.2B
Worksheet 2.2C

You may want to give

Classroom Experiment
In this small-group activity, students mark off the jet routes on the classroom floor or on an outdoor area. Students assume the roles of pilots, air traffic controllers, and NASA scientists. The pilots step down the jet routes at a prescribed pace. The NASA scientists track and record the pilots' times and the pilots' distances from the intersection of the routes. The air traffic controllers set the pace and measure the separation distance when the first plane arrives at the intersection.

Materials
Activity 2.2A: Set Up the Experiment
-- sidewalk chalk or masking tape
--measuring tape or ruler
--marking pens (optional)

students an overview of the experiment including an explanation of what they will do in each activity.

You may want to ask your students to compare the experiment distances and speeds with the real-world speeds given in the sidenote for Activity 2.0.

You may want to ask your students to estimate the route layout before they measure.

Students who have little experience in measurement may benefit from first practicing skip counting (by 6) to prepare them to measure 6-inch lengths.

It may be difficult for some student pilots to take 6-inch steps by placing one foot in front of the other. Instead, advise the pilots to place one foot on either side of the jet route and align their toes at each mark. It may be helpful for students to practice.

Activity 2.2B: Conduct the Experiment
- --1 stopwatch or 1 watch with a sweep second hand or 1 digital watch that indicates seconds
- --pencils and Data Sheets (Worksheet 2.2C)
- --signs identifying pilots, controllers, and NASA scientists
  Note: the signs are available on the Smart Skies website.
- --clipboard (optional)

Student Handouts:
--Worksheet 2.2A: Set Up the Experiment
--Worksheet 2.2B: Conduct the Experiment
--Worksheet 2.2C: Data Sheet

Worksheet 2.2A, Set Up the Experiment

If there is not enough room to set up two routes (a 20-foot route and a 16-foot route) at right angles to one another, another angle may be used. As an alternative, the routes may be set up parallel to each other. (Caution: parallel routes may confuse students who have not had much experience with the experiment. They may not make the connection between the parallel routes and the given merging routes.) In any case, allow enough distance between the routes so that the two pilots are not distracted by one another.

You may want to set up one pair of jet routes as a model that your students can copy.

After a group of students has completed its jet route set-up, you may find it helpful to have them compare their work with another student set-up.

Worksheet 2.2B, Conduct the Experiment

Assign students to positions on 6-8 person teams as follows:

- --Lead Air Traffic Controller (1 student)
- --Secondary Air Traffic Controller (1 student)
- --Pilots (2 students)
- --NASA Scientists, 1 or 2 for each plane (2 – 4 students)

After the jet routes are set up, have one group of students demonstrate the experiment while the rest of the class observes. Discuss and address any issues that may arise.

Perform the activity at least three times. Compare the results of each trial. Discuss the validity of the results.

Extensions:

1. Repeat the activity using different students as the Air Traffic Controllers, Pilots, and NASA Scientists.

2. Repeat the activity using jet routes longer than 20 feet and 16 feet, respectively. Keep the ratio of the lengths 20 : 16 (that is, 5 : 4). Increase the plane speed and the step size to 1 foot/second.

3. Have students draw a scale model of the experiment using real-world data. (See the sidenote for Activity 2.0).

Activity 2.3 --

Calculations

Estimated time:
15 - 30 minutes per worksheet

Calculate the Time for Each Plane to Reach the Intersection

This activity presents six different methods students can use to determine the number of seconds for each plane to arrive at the point where their routes merge.

Each worksheet may be assigned as either an individual or a small-group activity.

You can choose to assign one, some, or all of the worksheets. If students have completed an earlier Curriculum Supplement, you may decide to focus on only one worksheet.

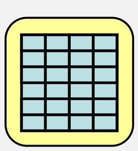
You may want to assign some worksheets before and some worksheets after the experiment.

The calculation methods range in order of difficulty as follows:

- Counting (completing a table)
- Drawing blocks to make a bar graph
- Plotting points on two vertical lines
- Plotting points on a Cartesian coordinate system
- Deriving and using the distance-rate-time formula
- Graphing two linear equations

Worksheet 2.3A, Count Feet and Seconds

Student Handout:
Worksheet 2.3A

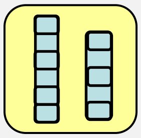


Students use patterns and skip-counting to complete a table and solve the problem. At the end of this activity, students may realize it is faster to multiply than to add to obtain the answer.

Prerequisite skills: count by 2s.

Worksheet 2.3B, Draw Blocks

Student Handout: Worksheet 2.3B



Students draw blocks, each representing the distance each plane travels in 10 seconds. The students "stack" their blocks along two vertical number lines (one line for each plane) that represent the jet routes.

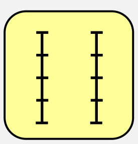
Notice that the vertical lines are numbered from 20 at the bottom to 0 at the top. Students begin to stack the blocks at the starting point of each plane, 20 feet away from the intersection and 16 feet away from the intersection, respectively. The intersection of the routes is represented with 0 at the top of each number line.

To help students make the connection between the "inverted Y" jet routes and the vertical scales, students are first asked to plot a point on the original jet route diagram and then stack the corresponding block along the vertical scale.

Prerequisite skills: read and build a bar graph with a vertical scale marked in 1-foot units; count by 10s.

Worksheet 2.3C, Plot Points on Two Vertical Lines

Student Handout: Worksheet 2.3C



This graph is similar to the way families record and compare the height of their children at the same ages. They mark off each child's birthday height (distance from the floor) on a doorway and then record their age (time since birth) beside the height mark.

The students plot their points along two vertical number lines (one line for each plane) that represent the jet routes.

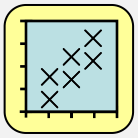
Notice that the vertical lines are numbered from 20 at the bottom to 0 at the top. The bottom of each number line corresponds to the starting point of the plane that is 20 feet away from the intersection. The intersection of the routes is represented with 0 at the top of each number line.

To help students make the connection between the "inverted Y" jet routes and the vertical scales, students are first asked to plot a point on the original jet route diagram and then plot the corresponding point on the vertical scale.

Prerequisite skills: plot a point on a (vertical) number line.

Worksheet 2.3D, Plot Points on a Cartesian Coordinate System

Student Handout: Worksheet 2.3D



Notice that the vertical axis is numbered from 0 at the top to **negative** 20 at the bottom. The numbers along the vertical axis represent the distance (with a negative sign attached) from the point where the two routes meet. Negative numbers are used because the points lie below the horizontal axis (the horizontal line at 0 feet).

Prerequisite skills: plot a point on a Cartesian coordinate system (the xy-plane)

Extension (optional):
For each plane, connect the points with a straight line. Find the equation of each line.

Student Handout: Worksheet 2.3E

d = r • t

Worksheet 2.3E, Derive the Distance-Rate-Time Formula

Students use patterns to derive the distance-rate-time formula in the form d = rt.

Prerequisite skills:
Use patterns to make a generalization.

Student Handout: Worksheet 2.3F

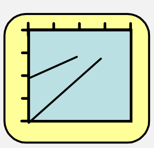
t = d / r

Worksheet 2.3F, Use the Distance-Rate-Time Formula

Students apply the distance-rate-time formula in the form t = d/r.

Prerequisite skills:
Substitute numbers into a formula.

Student Handout: Worksheet 2.3G



Worksheet 2.3G, Graph Two Linear Equations

Notice that the points are plotted in the fourth quadrant. So the given portion of the y- axis is numbered from 0 at the top to **negative** 20 at the bottom. The numbers along the y-axis represent the distance (with a negative sign attached) from the point where the two routes meet. Negative numbers are used because the points lie below the x-axis.

Prerequisite skills:
Graph a linear equation by making a table of ordered pairs.
Find the slope of a line given the equation of the line and the graph of the line.

Caution: Students may confuse the path of a plane with the graph of the plane's distance from the intersection as a function of time. In particular, students need to understand that the

Extension (optional):
You may want to ask your students to find the intercepts of each

routes meet, but the planes do not necessarily meet.

line and interpret those intercepts in the context of the airspace problem.

For each plane, the y-intercept represents the plane's initial distance (with negative sign attached) from the intersection point.

For each plane, the x-intercept represents the number of seconds for the plane to reach the intersection point.

The horizontal distance between the x-intercepts represents the number of seconds between the arrival of the two planes at the intersection.

Materials

Worksheet 2.3A: Calculate the time—count feet & seconds
Worksheet 2.3B: Calculate the time—draw blocks
Worksheet 2.3C: Calculate the time—plot on two vertical scales
Worksheet 2.3D: Calculate the time—plot points on a Cartesian coordinate system
Worksheet 2.3E: Derive the Distance-rate-time formula
Worksheet 2.3F: Use the Distance-rate-time formula
Worksheet 2.3G: Graph Two Linear Equations

Activity 2.4 --

Analysis

Estimated time:
45 minutes

Student Handout:
Worksheet 2.4

Compare the Experimental Results with the Predicted Results
Students compare the outcome of the experiment with their pretest predictions.

This activity may be assigned as either an individual or a small-group activity.

If you assigned some calculation worksheets (Activity 2.3) prior to the experiment, students can compare their calculations with the experimental results.

You may want to assign some Activity 2.3 calculation worksheets after the experiment to give students another basis for comparison.

As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to a movie theater. (See the Airspace Scenario section of this document.)

Now, you might suggest they consider two cars traveling in parallel lanes on the same road, with the two lanes merging into one lane. Each car is traveling at the same constant (fixed) speed. The cars are each a different distance from the merge. Students should realize that the cars will arrive at the merge at different times.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

Materials
Worksheet 2.4: After the Experiment

Activity 2.5 --

Posttest

Estimated time:
15 - 30 minutes

The posttest is ***optional.***

Student Handouts:
Worksheet 2.5
Worksheet 2.1B

Curriculum Supplement Posttest

This activity may be assigned as either an individual or a small-group activity.

You can direct your students to use a particular calculation method or methods to answer the posttest questions. Worksheet 2.1B (used for the Pretest) contains blank vertical line plots and grids that students can use as they do their calculations.

Materials
Worksheet 2.5: Posttest
Worksheet 2.1B: Lines and Grids

Name

Problem Statement

In the picture below, two airplanes are flying on different routes.

The World Airlines plane has flight number **WAL**27.

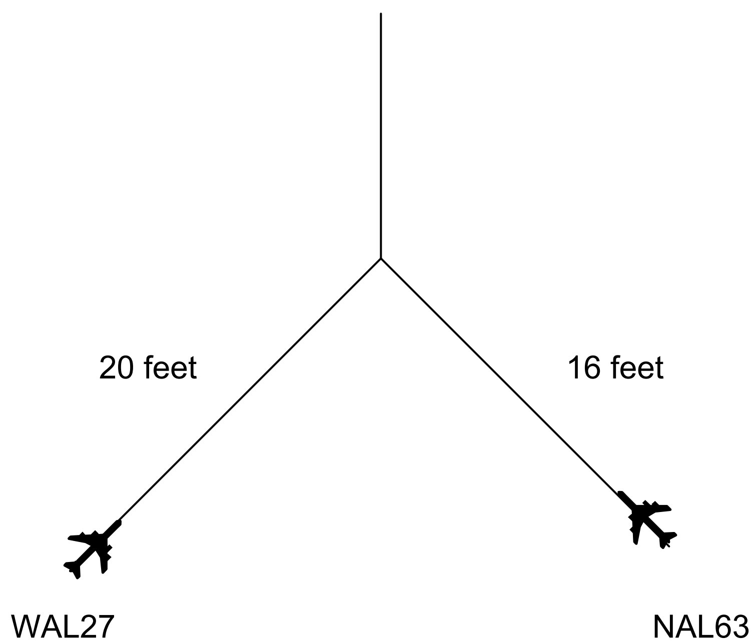
The speed of Flight WAL27 is 1/2 foot/second (0.15 meters/second).

The National Airlines plane has flight number **NAL**63.

The speed of Flight NAL63 is 1/2 foot/second (0.15 meters/second).

Flight WAL27 is 20 feet (6.1 meters) away from the point where the two routes come together.

Flight NAL63 is 16 feet (4.9 meters) away from the point where the two routes come together.



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Name

Question 1: How many seconds will it take Flight WAL27 to travel 20 feet to the point where the two routes come together?

Question 2: How many seconds will it take Flight NAL63 to travel 16 feet to the point where the two routes come together?

Question 3: Will the planes meet at the point where the two routes come together?

Question 4: If not, how many feet apart will the planes be when the first plane reaches the point where the routes come together?

Name

Pretest—Make a Prediction

In the picture below, two airplanes are flying on different routes.

1. Draw a circle around the point where the routes come together.

The World Airlines plane has flight number WAL27.

The speed of Flight WAL27 is 1/2 foot/second.

2. Write the speed of Flight WAL27 next to its picture.
3. How far does Flight WAL27 travel in one second? ________________

The National Airlines plane has flight number NAL63.

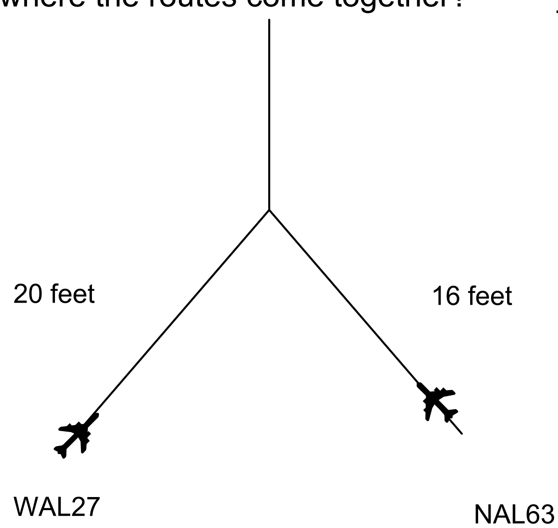
The speed of Flight NAL63 is 1/2 foot/second.

4. Write the speed of Flight NAL63 next to its picture.
5. How far does Flight NAL63 travel in one second? ________________
6. Do you think that the two planes will meet at the point where the two routes come together? ________________

   Why or why not? ____

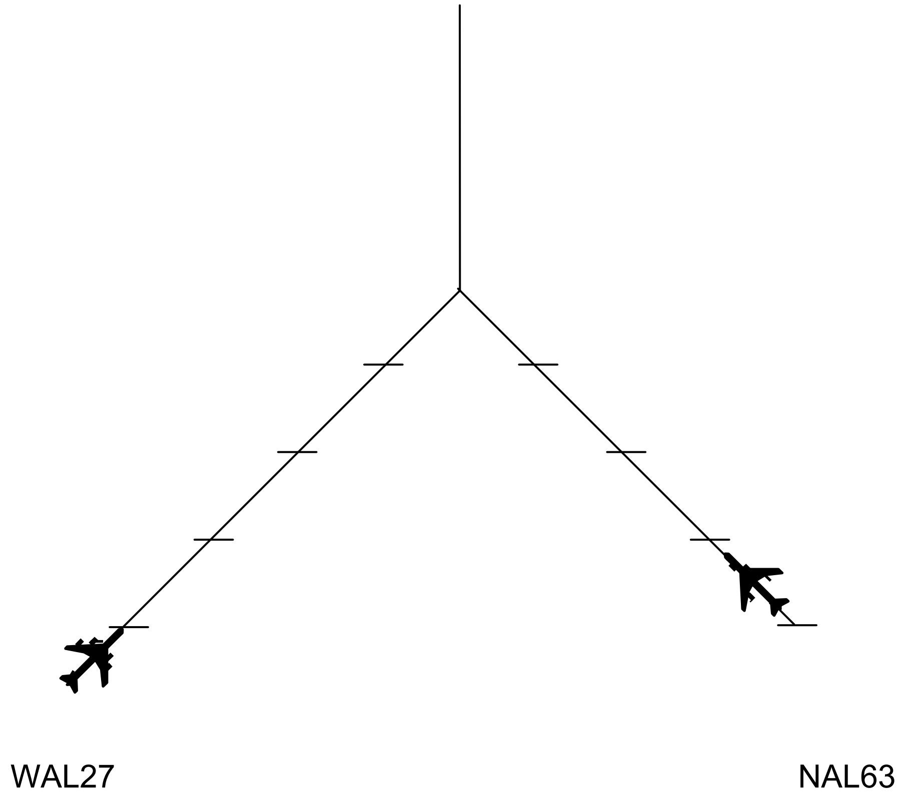
   ____

7. If not, how many feet apart do you think the planes will be when the first plane reaches the point where the routes come together? ________________



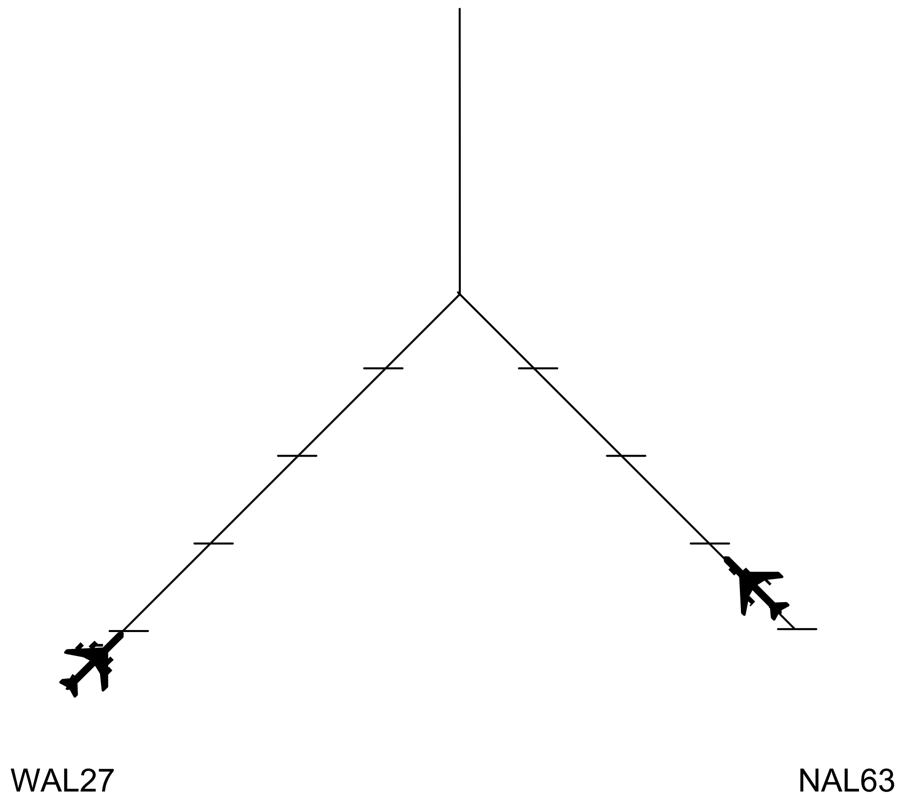
Name

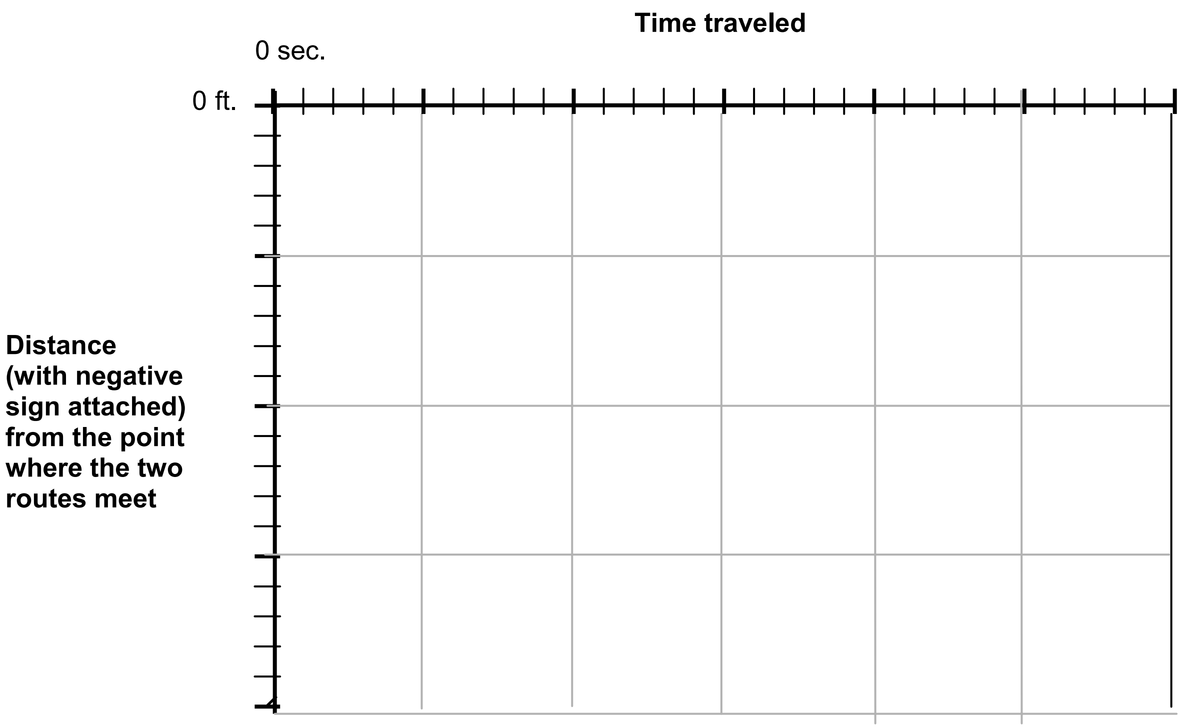
Lines and Grids



Name

Lines and Grids





____
Name

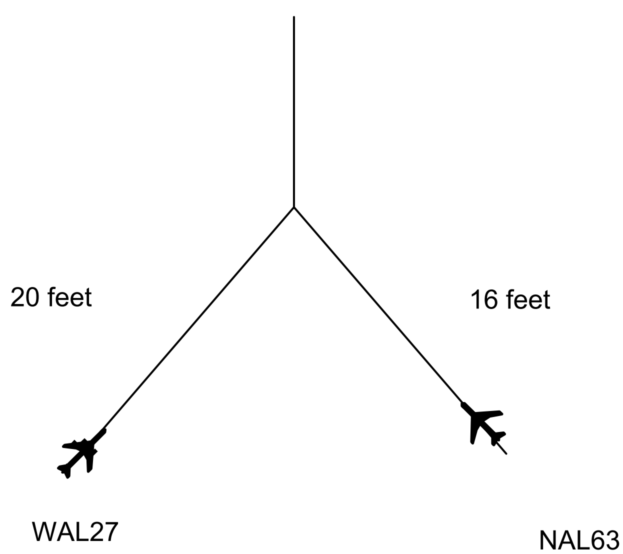
Set Up the Experiment

1. Use sidewalk chalk (or masking tape) to lay out 2 jet routes, one for each airplane as shown below.
   The route for WAL27 should be 20 feet long. The route for NAL63 should be 16 feet long.

2. The speed of Flight WAL27 is 1/2 foot/second. Stand at the beginning of the jet route for Flight WAL27. Place a mark (or a piece of masking tape) every 1/2 foot (every 6 inches) along the jet route. This will guide the pilot as he or she steps down the jet route.

3. The speed of Flight NAL63 is 1/2 foot/second. Stand at the beginning of the jet route for Flight NAL63. Place a mark (or a piece of masking tape) every 1/2 foot (every 6 inches) along the jet route. This will guide the pilot as he or she steps down the jet route.

4. On each jet route, place and label a longer chalk mark (or longer piece of masking tape) at the following positions:
   5 feet from the start, **10 feet** from the start, **15 feet** from the start, the **finish point**
   Note: The finish point is where the jet routes meet.



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Name

Conduct the Experiment

1. **Review your prediction.**

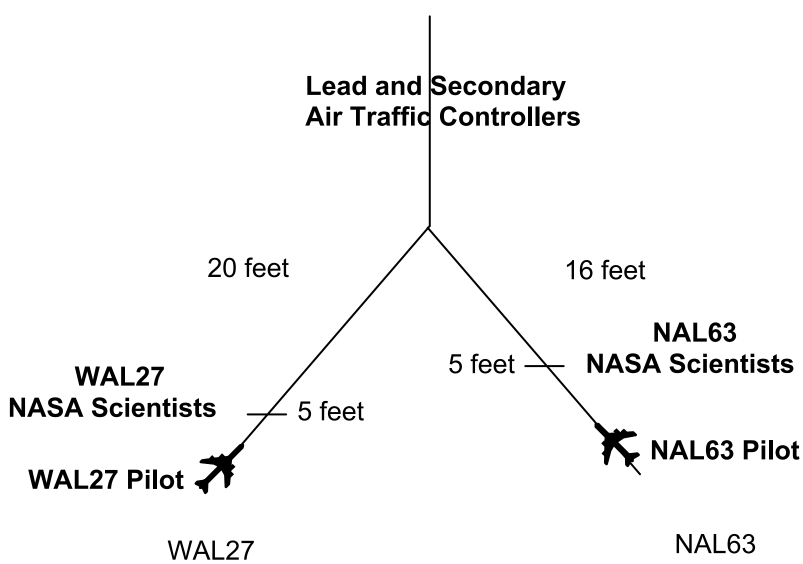
   Do you think the airplanes will meet at the point where the two routes meet? Why or why not?

2. **Take your position.** Circle your role in the diagram and in the following list:

   Lead Air Traffic Controller: Give the command "Take your ready positions."
   Pilots: Position yourself at the start of your jet route.
   Secondary Controllers: Take your data sheet, measuring tape, and pencil and go to your controller location as shown below.
   NASA Scientists: Take your data sheet and pencil and go to your observation position at the 5-foot line as shown below.

3. **Get ready to begin.** Circle your role in the following list:

   Lead Air Traffic Controller: Give the command "Set."
   Pilots: Prepare to step down your jet route. You may want to practice first. It helps to keep one foot on each side of the jet route.
   NASA Scientists: Get ready to measure and record the information on the data sheet.



____
Name

4. **Begin the experiment.** Circle your role in the following list:

   Lead Air Traffic Controller: Give the command "Ready." Start your stopwatch and count the seconds aloud, "One, two, three…" and so on.

   Pilots: Take your first step on count "One." Each second, take one step to the next timing mark.

   NASA Scientists: Record the time your aircraft arrives at the 5-foot line. Stay ahead of the pilot and record the time your aircraft arrives at the 10-foot line, the 15-foot line, and the point where the Controller says, "Halt."

5. **End the experiment.** Circle your role in the following list:

   Secondary Controller: When the first Pilot reaches the point where the two routes meet, give the command "Halt." Measure and record the separation distance between the planes. To do this, measure the distance of the second Pilot from the point where the two routes meet.

   Lead Air Traffic Controller: Stop counting the seconds when you hear "Halt."

   Pilots: Stop and remain where you are on the jet route when you hear "Halt."

   NASA Scientists: Record the "Halt" time.

Name: ____________________

Data Sheet

a. Fill in this table:

Flight Number	Speed	Distance from the Point Where the Two Routes Meet
WAL27		
NAL63		

b. On the picture below, circle your job title. Notice the data you need to record.

c. During Experiments 1, 2, and 3, record your data. **1 | 2 | 3**

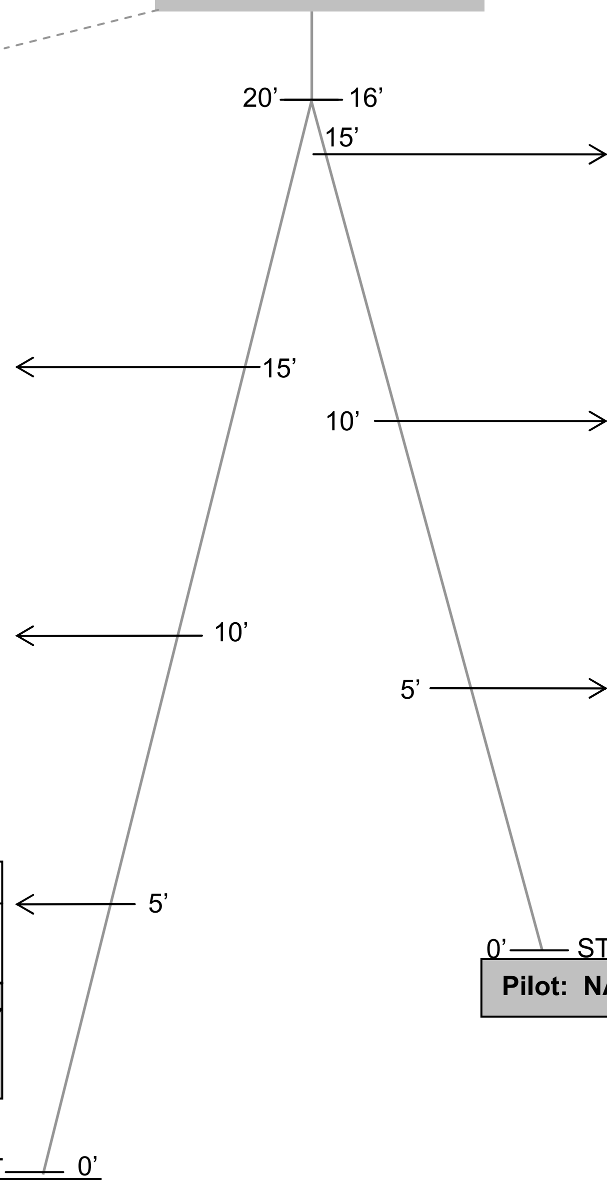
Separation Distance (feet) at Intersection

1	2	3

Air Traffic Controllers

"HALT" Time

1	2	3



WAL27 (left side)

"HALT" Time

1	2	3

Time (seconds) — 15'

1	2	3

Time (seconds) — 10'

1	2	3

Time (seconds) — 5'

1	2	3

NASA Scientist: WAL27
Start recording here.

Pilot: WAL27

NAL63 (right side)

Time (seconds) — 15'

1	2	3

Time (seconds) — 10'

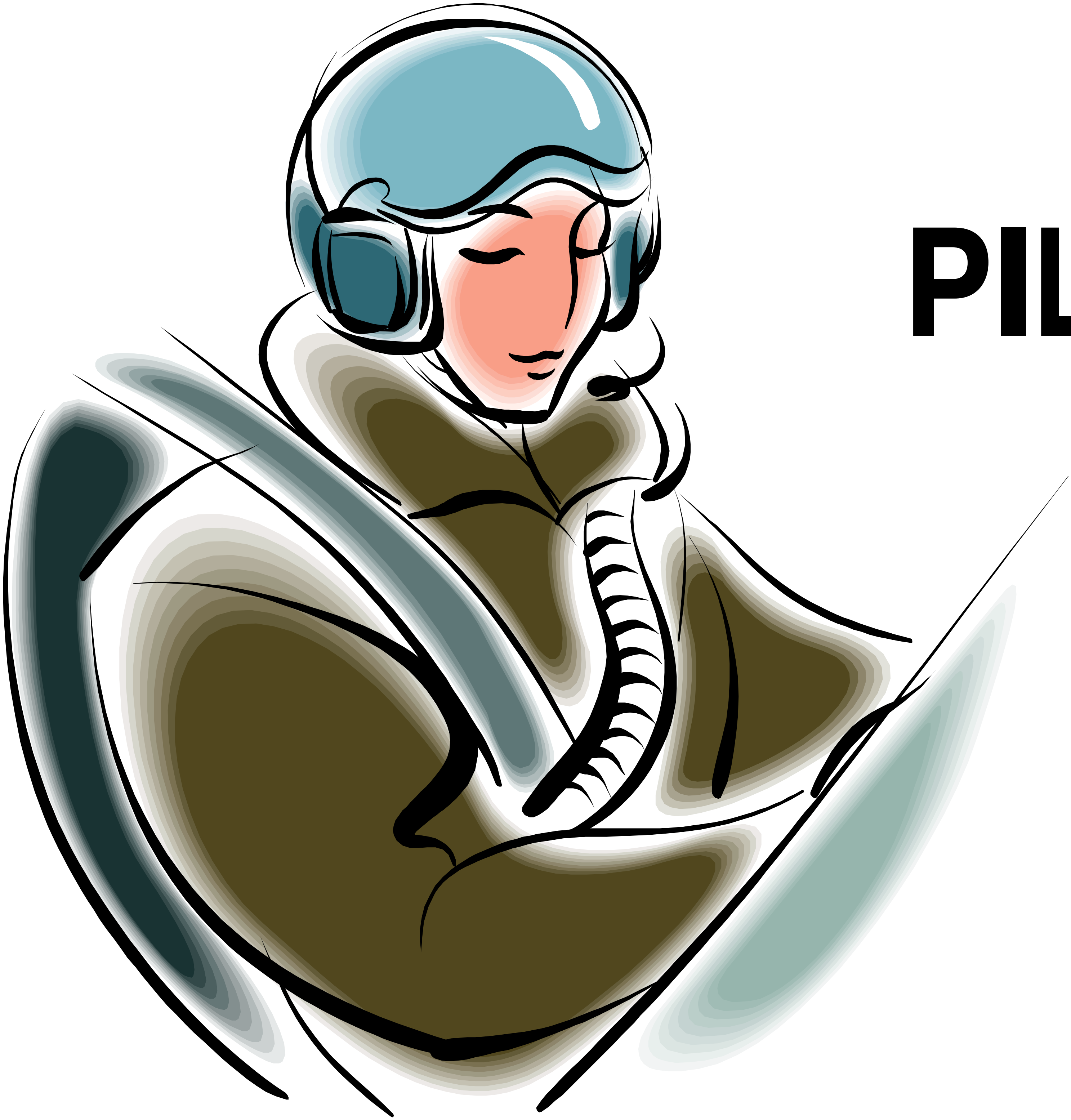
1	2	3

Time (seconds) — 5'

1	2	3

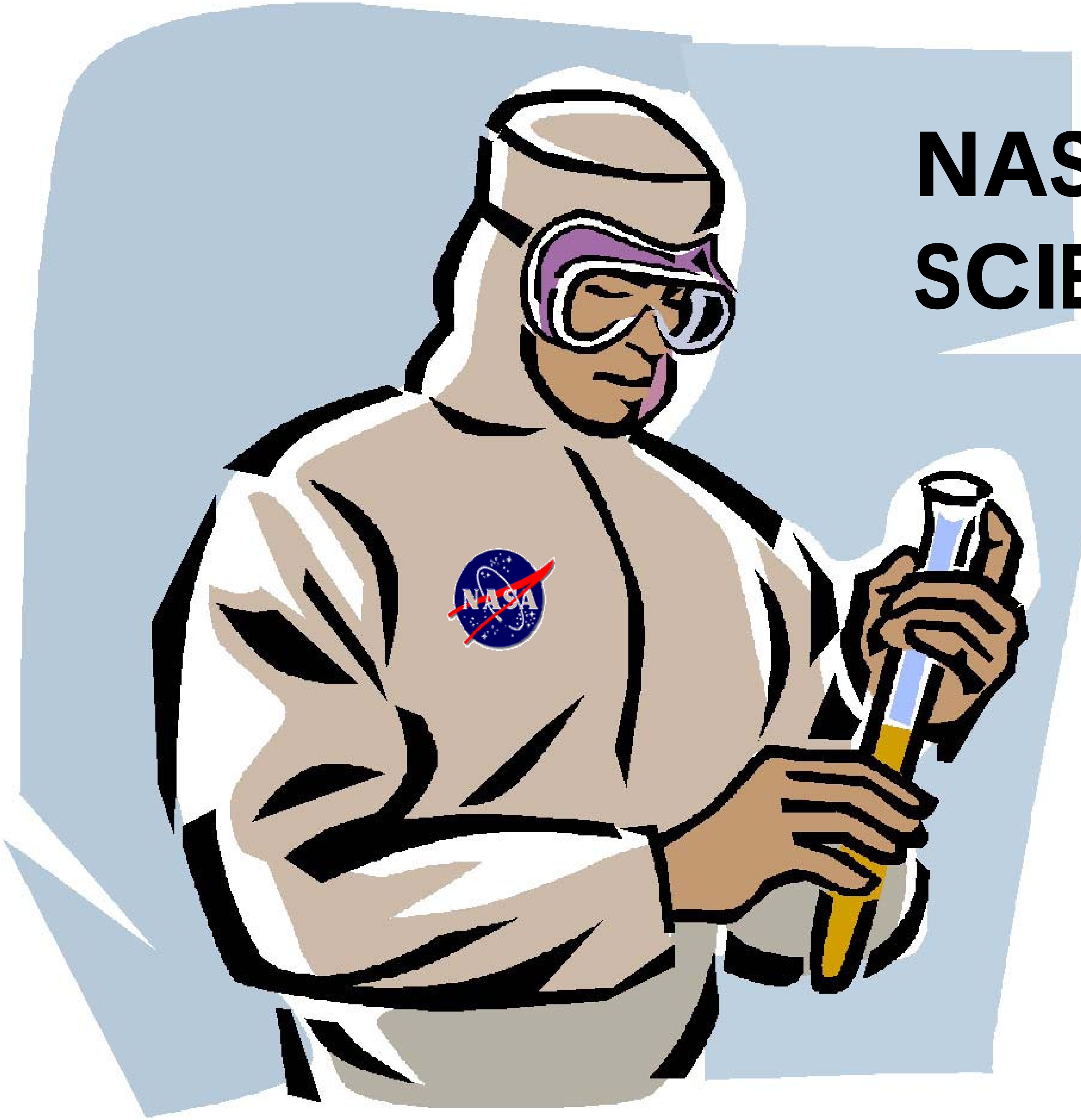
NASA Scientist: NAL63
Start recording here.

Pilot: NAL63



PILOT

AIR TRAFFIC CONTROLLER



NASA SCIENTIST

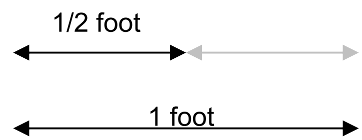
Name

How Much Time To Reach the Point Where the Two Routes Meet?

(Count Feet and Seconds to Find the Answer)

The speed of each airplane is 1/2 foot per second.

That means each airplane travels 1/2 foot in 1 second.



So each airplane travels 1 foot in 2 seconds.

Flight WAL27 starts 20 feet from the point where the two routes meet.

Flight NAL63 starts 16 feet from the point where the two routes meet.

1. Fill in the given table to see how many seconds it will take each plane to travel to the point where the two routes meet.

After you fill in the table, answer the following questions:

2. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 ____________ seconds NAL63 ____________ seconds

3. Will the two planes meet at the point where the two routes come together? ________________

4. Why or why not? ____

   ____

5. You filled in the table to find the answer. Can you think of a faster way to find the answer? If so, describe the faster way. ________________________________

   ____

6. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? ____________________________________

   ____

Name

Flight WAL27		Flight NAL63	
How many feet?	How many seconds?	How many feet?	How many seconds?
1	2	1	2
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

Name

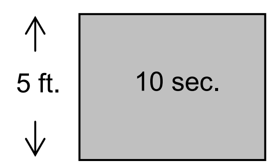
How Much Time To Reach the Point Where the Two Routes Meet?
(Draw Blocks to Find the Answer)

Flight WAL27 starts 20 feet from the point where the two routes meet.
Flight NAL63 starts 16 feet from the point where the two routes meet.

The speed of each airplane is 1/2 foot per second.
In 1 second, each airplane travels 1/2 foot.
In 2 seconds, each airplane travels 1 foot.

In 10 seconds (5 x 2 seconds), each plane travels 5 x 1 foot.
That is, each plane travels 5 feet in 10 seconds.

The height of this block represents 5 feet, the distance each plane travels in 10 seconds.



Now you will use blocks and dots to plot the position of each plane as it travels to where the two routes meet.

Look at the top picture on page 4. The picture shows the two jet routes.

Flight NAL63 started 16 feet away from the point where the two routes meet.
After 10 seconds, Flight NAL63 has moved 5 feet closer to that point.
So the plane is 11 feet from the point where the two routes meet.
A dot shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 started 20 feet away from the point where the two routes meet.
After 10 seconds, Flight WAL27 has moved 5 feet closer to that point.
So the plane is 15 feet from the point where the two routes meet.
A dot shows the position of Flight WAL27 after 10 seconds.

Name

The dots are connected with a line marked "10 seconds."

Next look at the bottom picture on page 4. The shows a bar graph made of blocks.

A block shows the position of Flight NAL63 after 10 seconds.
A block shows the position of Flight WAL27 after 10 seconds.

The blocks are connected with a line marked "10 seconds."

Now it's your turn to draw and connect.

- ◻ On the top picture on page 4, draw a dot to show the position of Flight NAL63 after 20 seconds.
- ◻ On the bottom picture on page 4, trace the block that shows the position of Flight NAL63 after 20 seconds.

- ◻ On the top picture, draw a dot to show the position of Flight WAL27 after 20 seconds.
- ◻ On the bottom picture, trace the block that shows the position of Flight WAL27 after 20 seconds.

- ◻ Connect your dots with a line marked "20 seconds."
- ◻ Connect your blocks with a line marked "20 seconds."

- ◻ Draw and connect dots at 30 seconds.
- ◻ Draw and connect blocks at 30 seconds.

- ◻ Keep going until the first plane reaches the place where the two routes meet.

____
Name

When you are done, answer these questions.

1. How many seconds will it take each plane to travel to the point where the two routes meet?

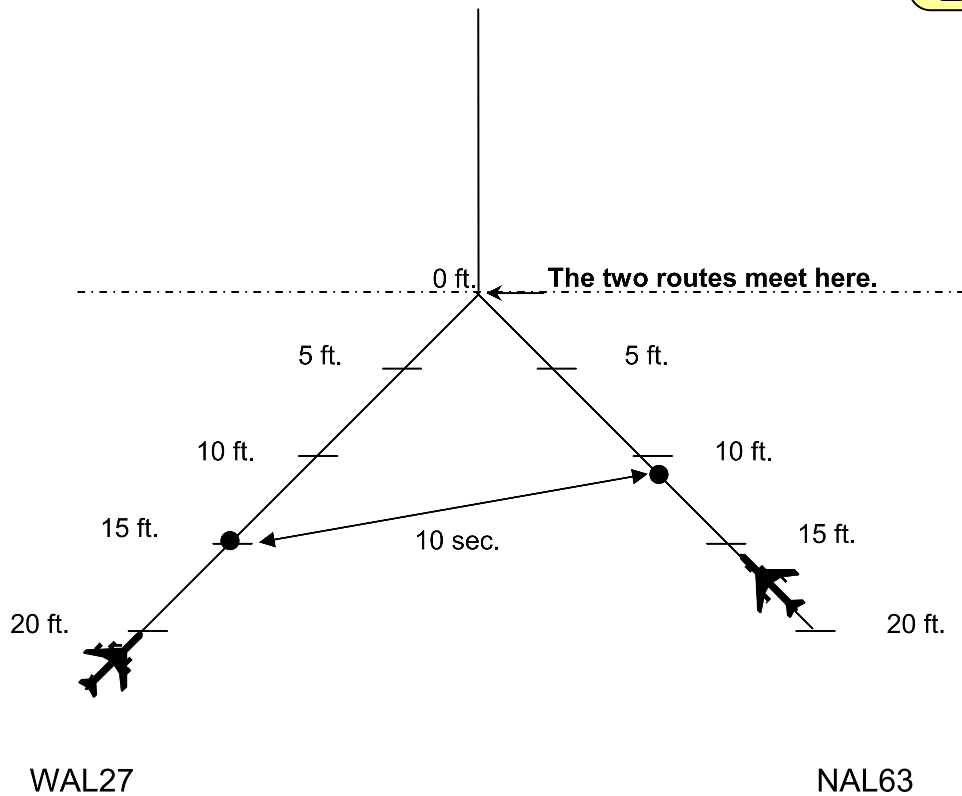
   WAL27 ____________ seconds NAL63 ____________ seconds

2. Will the two planes meet at the point where the two routes come together? ________________

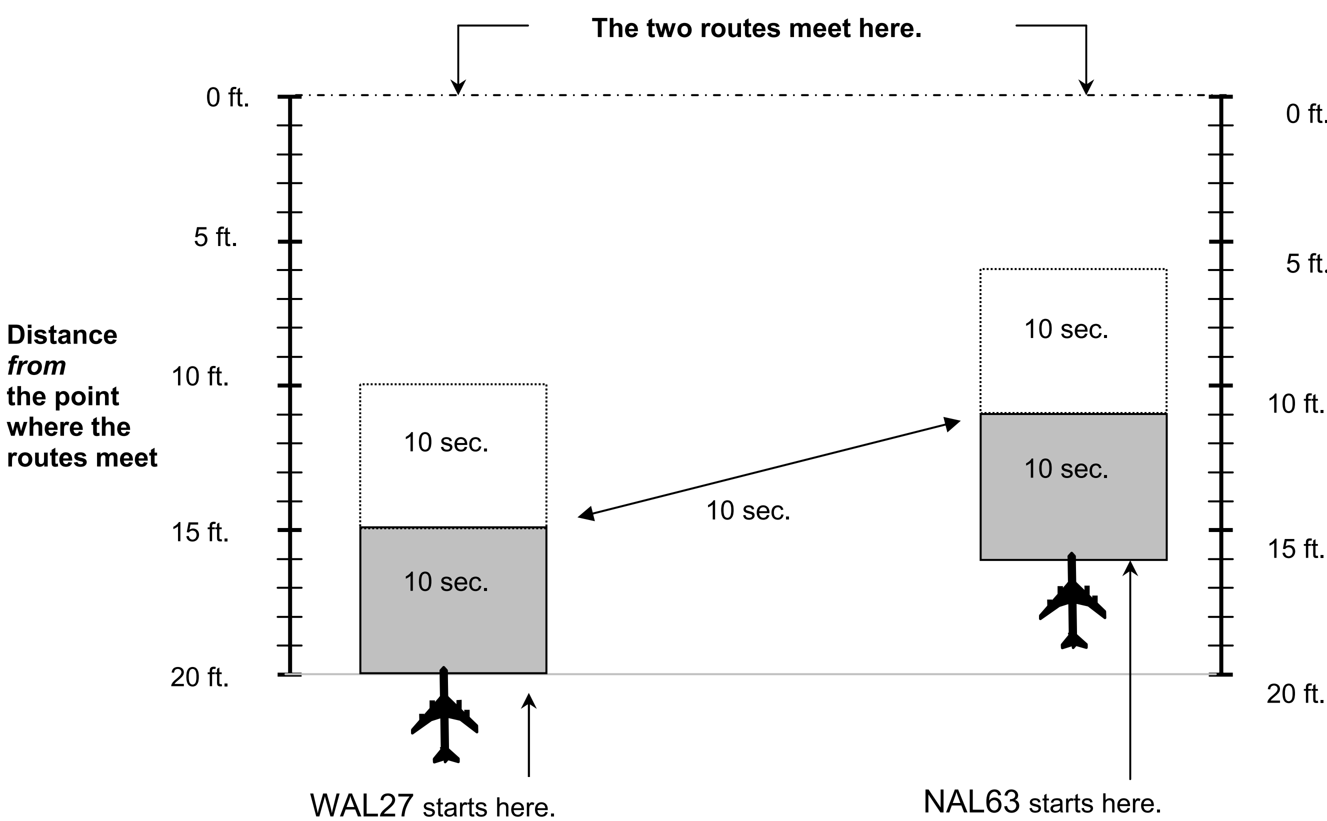
3. Why or why not? ____

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? ______________________________________

Name



0 ft.
The two routes meet here.
5 ft.
5 ft.
10 ft.
10 ft.
15 ft.
10 sec.
15 ft.
20 ft.
20 ft.
WAL27
NAL63



The two routes meet here.
0 ft.
0 ft.
5 ft.
5 ft.
Distance *from* the point where the routes meet
10 ft.
10 sec.
10 ft.
10 sec.
10 sec.
10 sec.
15 ft.
10 sec.
15 ft.
20 ft.
20 ft.
WAL27 starts here.
NAL63 starts here.

Name

How Much Time To Reach the Point Where the Two Routes Meet?
(Plot Points on Lines to Find the Answer)

Flight WAL 27 starts 20 feet from the point where the two routes meet.
Flight NAL63 starts 16 feet from the point where the two routes meet.

The speed of each airplane is 1/2 foot per second.

In 1 second, each airplane travels 1/2 foot.
In 2 seconds, each airplane travels 1 foot.

In 10 seconds (5 x 2 seconds), each plane travels 5 x 1 foot.
That is, each plane travels 5 feet in 10 seconds.

You will use an **O** to show the position of Flight NAL63.
You will use an **X** to show the position of Flight WAL27.

Look at the top picture on page 4. The picture shows the two jet routes.

Flight NAL63 started 16 feet away from the point where the two routes meet.
After 10 seconds, Flight NAL63 has moved 5 feet closer to that point.
So the plane is 11 feet from the point where the two routes meet.
An **O** shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 started 20 feet away from the point where the two routes meet.
After 10 seconds, Flight WAL27 has moved 5 feet closer to that point.
So the plane is 15 feet from the point where the two routes meet.
An **X** shows the position of Flight WAL27 after 10 seconds.

The **X** and **O** are connected with a line marked “10 seconds.”

Name

Next look at the bottom picture on page 4. It shows two vertical line graphs.

An **O** shows the position of Flight NAL63 after 10 seconds.
An **X** shows the position of Flight WAL27 after 10 seconds.

The **X** and **O** are connected with a line marked "10 seconds."

Now it's your turn to draw and connect.

- ◻ On the top picture, draw an **O** to show the position of Flight NAL63 after 20 seconds.
- ◻ On the bottom picture, draw an **O** to show the position of Flight NAL63 after 20 seconds.

- ◻ On the top picture on page 4, draw an **X** to show the position of Flight WAL27 after 20 seconds.
- ◻ On the bottom picture on page 4, draw an **X** to show the position of Flight WAL27 after 20 seconds.

- ◻ On the top picture, connect the **X** and **O** with a line marked "20 seconds."
- ◻ On the bottom picture, do the same thing.

- ◻ On the top picture, draw and connect an **X** and an **O** at 30 seconds.
- ◻ On the bottom picture, do the same thing.

- ◻ Keep going until the first plane reaches the place where the two routes meet.

____
Name

When you are done, answer these questions.

1. How many seconds will it take each plane to arrive at the point where the two routes meet?

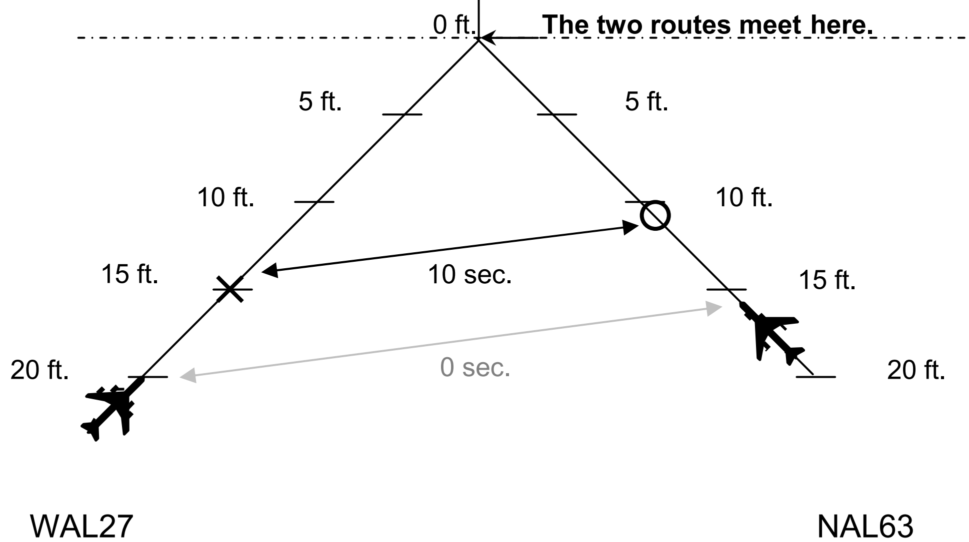
   WAL27 ___________ seconds NAL63 ___________ seconds

2. Will the two planes meet at the point where the two routes come together? _______________

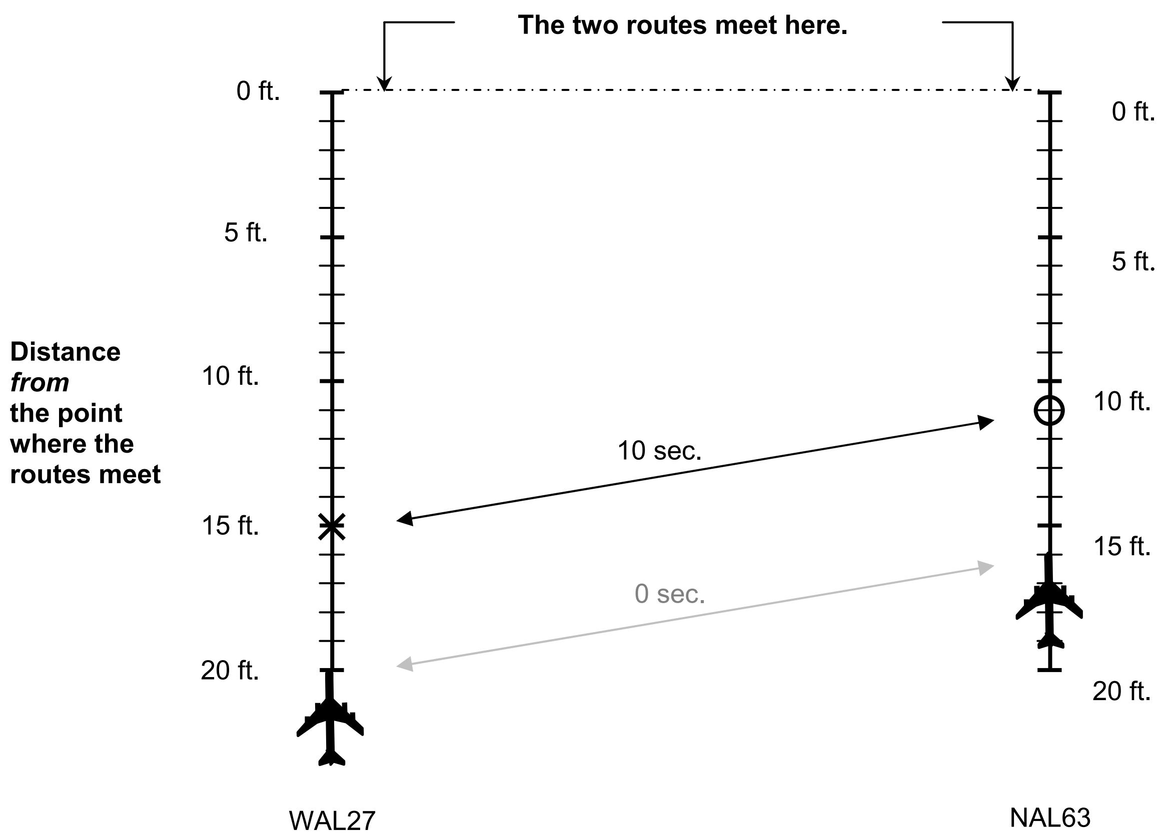
3. Why or why not? ____

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? __________________________________

Name



0 ft.
The two routes meet here.
5 ft.
5 ft.
10 ft.
10 ft.
15 ft.
10 sec.
15 ft.
20 ft.
0 sec.
20 ft.
WAL27
NAL63



The two routes meet here.
0 ft.
0 ft.
5 ft.
5 ft.
Distance *from* the point where the routes meet
10 ft.
10 ft.
10 sec.
15 ft.
15 ft.
0 sec.
20 ft.
20 ft.
WAL27
NAL63

Name

How Much Time To Reach the Point Where the Routes Meet?
(Plot Points on a Grid to Find the Answer)

The speed of each airplane is 1/2 foot per second.
In 2 seconds, each airplane travels 1 foot.
In 10 seconds, each plane travels 5 feet.

Flight NAL63 starts 16 feet from the point where the two routes meet.
After 10 seconds, the plane has traveled 5 feet.
So after **10** seconds, Flight NAL63 is **11** feet ***from*** the point where the two routes meet.

On the NAL63 jet route on page 3, we represent that information with an **O** at 11 feet and an arrow labeled “10 seconds.”

On the grid on page 3, we represent that information with the point (**10**, **-11**).
We use **negative** eleven because the point lies **below** the horizontal line at 0 feet where the two routes meet.
The **O** at the point (10, -11) shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 starts 20 feet from the point where the two routes meet.
After 10 seconds, the plane has traveled 5 feet.
So after **10** seconds, Flight WAL27 is **15** feet ***from*** the point where the two routes meet.

On the WAL27 jet route, we represent that information with an **X** at 15 feet and an arrow labeled “10 seconds.”

On the grid, we represent that information with the point (**10**, **-15**).
We use **negative** fifteen because the point lies **below** the horizontal line at 0 feet where the two routes meet.
The **X** at the point (10, -15) shows the position of Flight WAL27 after 10 seconds.

____
Name

Now it's your turn to plot and connect points on the routes and to plot points on the grid on page 3.

Put an **O** at the point that shows the position of Flight NAL63 after 20 seconds, 30 seconds and so on.

Put an **X** at the point that shows the position of Flight WAL27 after 20 seconds, 30 seconds, and so on.

Keep going until the first plane reaches the horizontal line at 0 ft.
That line represents the point where the two routes meet.

When you have finished plotting points, answer these questions.

1. How many seconds will it take each plane arrive at the point where the two routes meet?

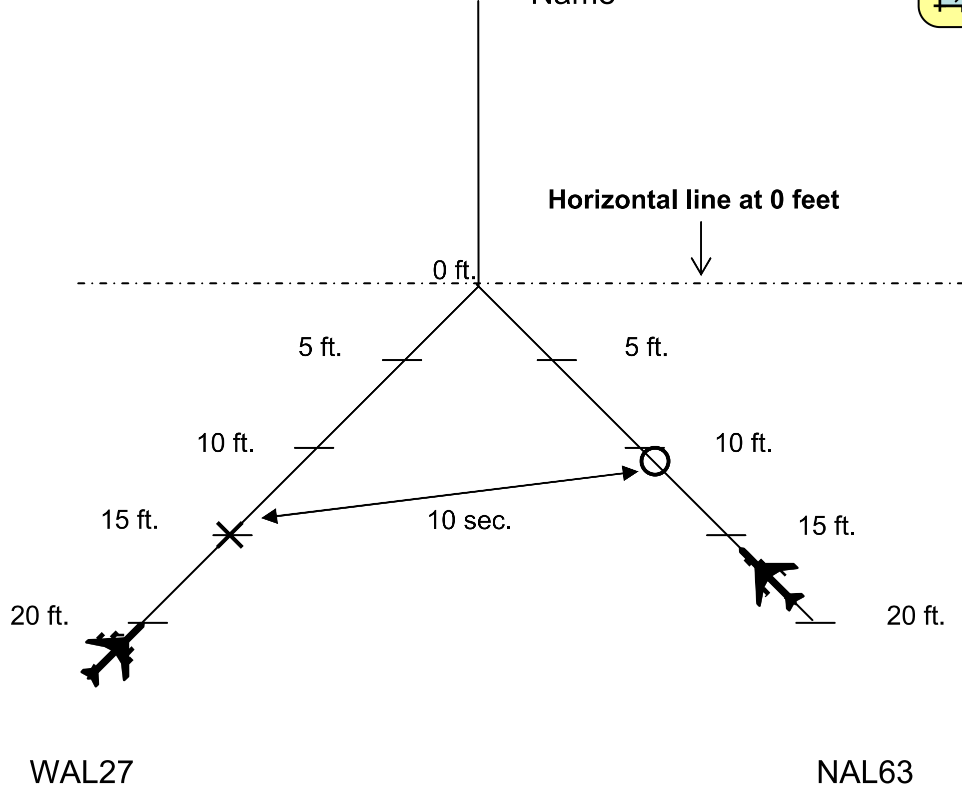
   WAL27 ____________ seconds NAL63 ____________ seconds

2. Will the two planes meet at the point where the two routes come together? ________________

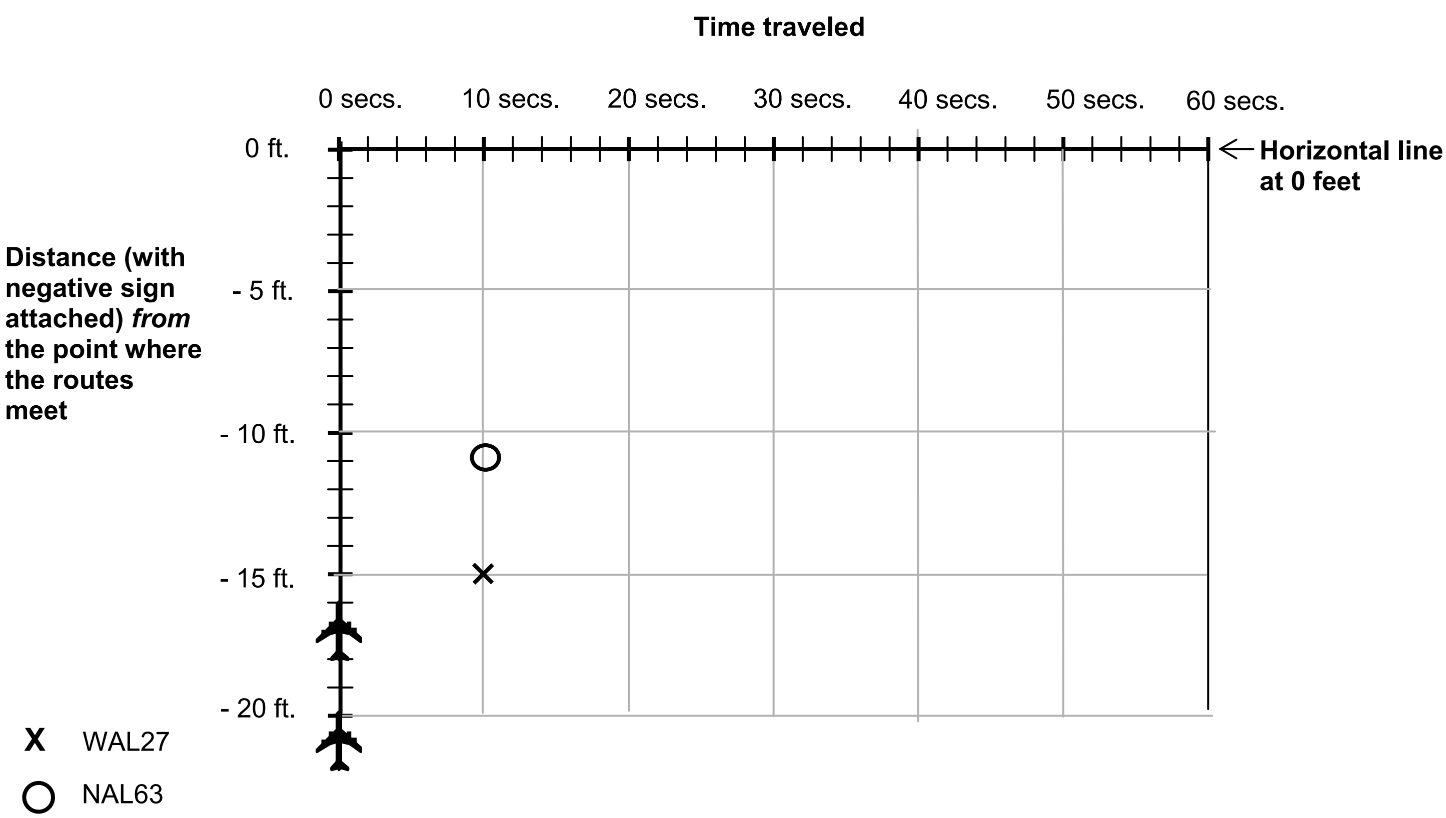
3. Why or why not? ____

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? ____

Name



Horizontal line at 0 feet
0 ft.
5 ft.
5 ft.
10 ft.
10 ft.
15 ft.
10 sec.
15 ft.
20 ft.
20 ft.
WAL27
NAL63



Time traveled
0 secs.
10 secs.
20 secs.
30 secs.
40 secs.
50 secs.
60 secs.
Horizontal line at 0 feet
0 ft.
- 5 ft.
- 10 ft.
- 15 ft.
- 20 ft.
Distance (with negative sign attached) *from* the point where the routes meet
X WAL27
O NAL63

____

Name

d = r • t

Derive the Distance-Rate-Time Formula

The speed of each airplane is 0.5 feet per second.

In 1 second, each plane travels 0.5 feet.

In 2 seconds, each plane travels 0.5 feet/second $\times$ 2 seconds = 1.0 foot.

In 3 seconds, each plane travels 0.5 feet/second $\times$ 3 seconds = 1.5 feet.

1. In 4 seconds, each plane travels _____________ $\times$ _________ = ____ feet.
2. In 5 seconds, each plane travels _____________ $\times$ _________ = ____ feet.
3. In 6 seconds, each plane travels _____________ $\times$ _________ = ____ feet.
4. How could you use multiplication to find the distance each plane travels in 14 seconds?

____

____

One way to find the distance is to multiply the plane's speed by 14 seconds, like this:

0.5 feet/second $\times$ 14 seconds = 7 feet

This suggests the following rule:

To find the distance traveled, multiply the speed by the time traveled.

In mathematics and science, we often say "rate" instead of "speed."

Then we can write the rule like this:

distance = **r**ate $\times$ **t**ime

This relationship is called the Distance-Rate-Time Formula. We often write it like this:

d = r • t

5. Use the formula to find the distance traveled by each plane in 20 seconds.

   In 20 seconds, each plane travels ___________ feet.

____
Name

t = d / r

Use the Distance-Rate-Time Formula

The speed of each airplane is 0.5 feet per second.

In 1 second, each plane travels 0.5 feet.

In 2 seconds, each plane travels 0.5 feet/second $\times$ 2 seconds = 1.0 foot.

In 3 seconds, each plane travels 0.5 feet/second $\times$ 3 seconds = 1.5 feet.

To find the distance traveled after t seconds, we multiply the rate by the time:

distance traveled = **r**ate of travel $\times$ **t**ime traveled

This relationship is called the Distance-Rate-Time Formula. We often write it like this:

$$\mathbf{d = r \bullet t}$$

If we divide both sides of this equation by r, we get a formula for time traveled:

$$\mathbf{t = \frac{d}{r}}$$

$$\frac{d}{r} = \frac{r \bullet t}{r}$$

You can use this formula to find the number of seconds for Flight WAL27 to travel 20 feet to the point where the two routes meet.

Flight WAL27 $t = \dfrac{\text{20 feet}}{\text{0.5 feet per second}}$ = ______ seconds

(Hint: Divide 20 by 0.5 .)

Use the same formula to find the number of seconds for Flight NAL63 to travel 16 feet to the point where the two routes meet.

Flight NAL63 ______ seconds

Name ____________________

t = d / r

Now answer these questions.

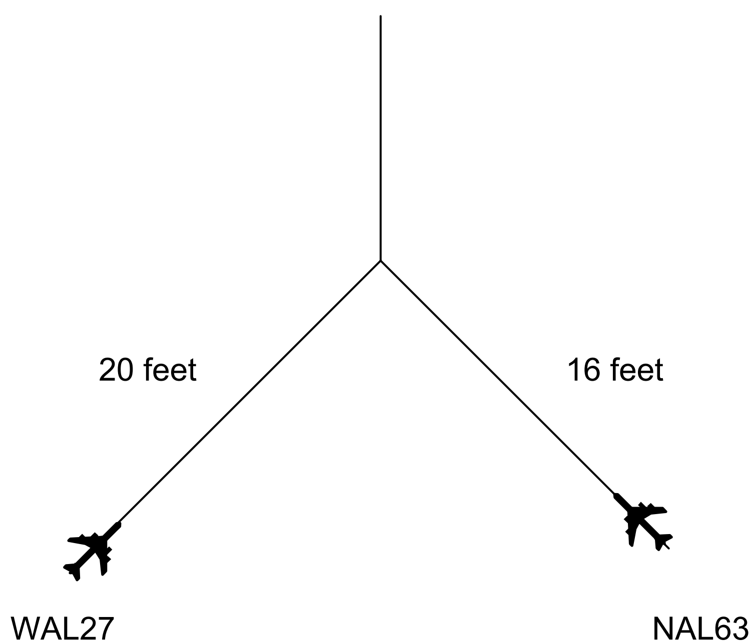
1. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 ____________ seconds NAL63 ____________ seconds

2. Will the two planes meet at the point where the two routes come together? ________________

3. Why or why not? ____

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? ________________________________



____
Name

How Much Time To Reach the Point Where the Two Routes Meet?
(Graph Two Linear Equations to Find the Answer)

We can use a linear equation to describe the position of an airplane that travels at a constant speed.

We begin with Flight WAL27.

The speed of Flight WAL27 is 0.5 feet per second.

When the clock starts at 0 seconds, the plane is 20 feet from the point where the two routes meet.

The position of Flight WAL27 is given by this equation:

$y = 0.5x – 20$ **WAL27**

Here:
x = the time traveled (in seconds) and
y = the number of feet (with a negative sign attached) from the intersection of the two routes.

Notice that when $x = 0$, $y = –20$.

This means that when the clock starts at 0 seconds, the plane is 20 feet from the point where the two routes meet.

Now think about Flight NAL63.

The speed of Flight NAL63 is also 0.5 feet per second. However, its distance from the intersection is 16 feet. So the position of Flight NAL63 is given by this equation:

$y = 0.5x – 16$ **NAL63**

Name

Complete each table and use the ordered pairs to graph each line.

WAL27 $y = 0.5x - 20$

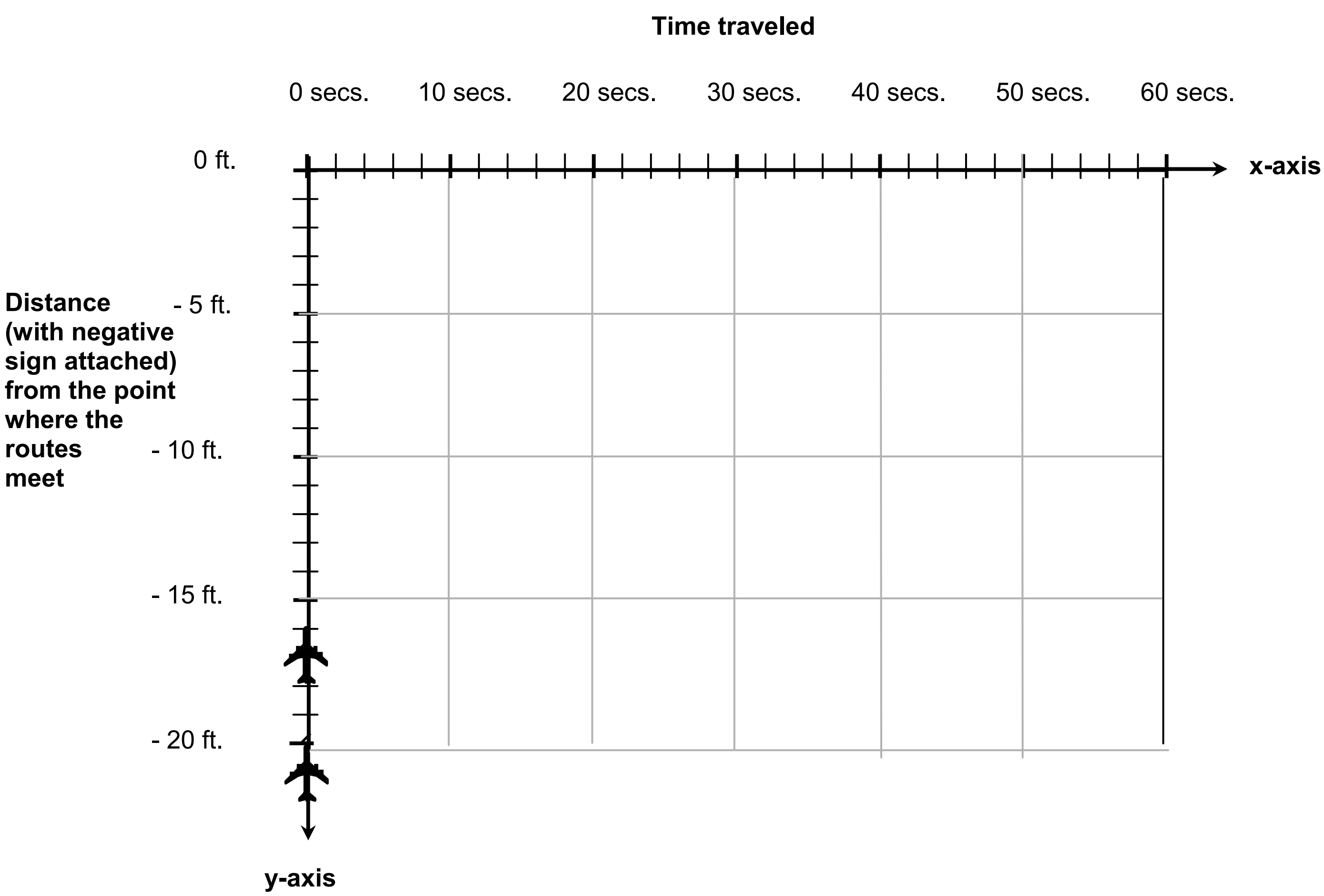
x	y
0	
10	
20	
30	

NAL63 $y = 0.5x - 16$

x	y
0	
10	
20	
30	

Use a solid line for the graph of **Flight WAL27**.

Use a dotted line for the graph of **Flight NAL63**.



____
Name

Use your graphs to answer the following questions.

1. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 ____________ seconds   NAL63 ____________ seconds

2. Will the two planes meet at the point where the two routes come together? ________________

3. Why or why not? ____

   ____

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? __________________________________

5. Write the number that is the slope of the solid line representing Flight WAL27. ___________

6. Write the number that is the slope of the dotted line representing Flight NAL63. ___________

7. What information does the slope of each line tell you about each plane?

   ____

   ____

____
Name

After the Experiment

Now you will compare your prediction with the results of the experiment.

First, circle your role in the experiment:

Pilot of WAL27	NASA Scientist for WAL27	Lead Air Traffic Controller
Pilot of NAL63	NASA Scientist for NAL63	Other Air Traffic Controller

----

Take a look at your prediction.
Did you predict the planes would meet at the point where the two routes come together? Yes No

Next look at the results of the experiment.
Did the planes meet at the point where the two routes come together? Yes No

Does your prediction match the experiment? Yes No

If your answer to the last question is No, why do you think your prediction and the experiment do not match? ____

____

Take another look at your prediction.

When the first plane gets to the point where the routes meet, how many feet away did you think the second plane would be?

Does your prediction match the experiment?

If your answer is No, why do you think your prediction and the experiment do not match? ____

____

Name

Take another look at the problem.

The speed of Flight WAL27 was 1/2 foot per second.

The speed of Flight NAL63 was 1/2 foot per second.

Were the speeds the same or different?

Same speed   Different speeds

Flight WAL27 started 20 feet from the point where the routes come together.

Flight NAL63 started 16 feet from the point where the routes come together.

Were the distances from the finish point the same or different?

Same distance   Different distances

Now think about this general problem.

Two planes are traveling at the same speed on two different routes.

The planes are different distances from the point where the two routes come together.

Will the planes meet at the point where the routes come together? ________________

If not, how far apart will the planes be when the first plane reaches the point where the routes come together?____

Explain your answers. ____

____

Name

Posttest

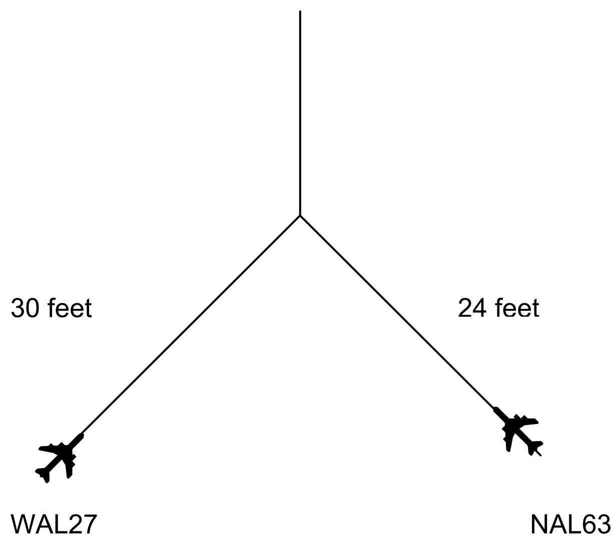
In the picture below, two airplanes are flying on different routes.

The speed of Flight WAL27 is 1 foot/second.

Flight WAL27 is 30 feet from the point where the two routes come together.

The speed of Flight NAL63 is 1 foot/second.

Flight NAL63 is 24 feet from the point where the two routes come together.



1. Do you think that the two planes will meet at the point where the two routes come together? ________________

   Why or why not? ____

   ____

2. If not, how far apart do you think the planes will be when the first plane reaches the point where the routes come together? ________________

____
Name

Now consider this general problem.

Two planes are traveling at the same speed on two different routes.
The planes are different distances from the point where the two routes come together.

3. Will the planes meet at the point where the routes come together?_____________

4. If not, how far apart will the planes be when the first plane reaches the point where the routes come together? ____

5. Explain your answers. ____
____

6. If you think the two planes would meet, what could you tell the air traffic controllers to do to avoid a collision? ____

Airspace Systems – Predicting Air Traffic Conflicts

Curriculum Supplement 2

ANSWERS & EXPLANATIONS

A note on the organization of the Answers and Explanations:
In this Curriculum Supplement, most of the activities pose the same set of questions. The answers to those questions are introduced in the first part of this answer document. The remainder of the document is organized by activity and includes answers to individual activity questions, graphs, applications of the distance-rate-time formula, and discussions of the general problems posed in the analysis activity and the posttest.

The speed of Flight WAL27 is 1/2 foot/second, so the plane travels 1/2 foot in 1 second.
The speed of Flight NAL63 is 1/2 foot/second, so the plane travels 1/2 foot in 1 second.
Flight WAL27 is 20 feet from the point of intersection.
Flight NAL63 is 16 feet from the point of intersection.

Since the planes are traveling at the same constant (fixed) speed and must travel a different distance to the point of intersection, a conflict will *not* occur at the intersection.

In particular:

- It will take 40 seconds for Flight WAL27 to travel 20 feet to the point where the routes come together.
- It will take 32 seconds for Flight NAL63 to travel 16 feet to the point where the routes come together.
- Since the times are different for each airplane, they will arrive at different times and a conflict will not occur.
- At 32 seconds, when Flight NAL63 arrives at the intersection, Flight WAL27 will be 4 feet from the intersection. So the planes will be 4 feet apart when the first plane arrives at the intersection.

Activity 2.3A—Count Feet and Seconds:

Each plane travels 1 foot in 2 seconds. Count by 2s to complete the table.

Flight WAL27 will travel 20 feet in 40 seconds. (Students can also multiply 2 seconds per foot by 20 feet to obtain 40 seconds.)
Flight NAL63 will travel 16 feet in 32 seconds. (Students can also multiply 2 seconds per foot by 16 feet to obtain 32 seconds.)
Since the times are different for each airplane, they will arrive at different times and a conflict will *not* occur.

1. Fill in the given table to see how many seconds it will take each plane to travel to the point where the two routes meet.

Flight WAL27		Flight NAL63	
How many feet?	How many seconds?	How many feet?	How many seconds?
1	2	1	2
2	**4**	**2**	**4**
3	**6**	**3**	**6**
4	**8**	**4**	**8**
5	**10**	**5**	**10**
6	**12**	**6**	**12**
7	**14**	**7**	**14**
8	**16**	**8**	**16**
9	**18**	**9**	**18**
10	**20**	**10**	**20**
11	**22**	**11**	**22**
12	**24**	**12**	**24**
13	**26**	**13**	**26**
14	**28**	**14**	**28**
15	**30**	**15**	**30**
16	**32**	**16**	**32**
17	**34**		
18	**36**		
19	**38**		
20	**40**		

Activity 2.3A (cont.)

2. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 **40** seconds NAL63 **32** seconds

3. Will the two planes meet at the point where the two routes come together? **No**

4. Why or why not? **At 32 seconds, when Flight NAL 63 arrives at the intersection, Flight WAL27 will be 4 feet from the intersection. So the planes will not meet.**

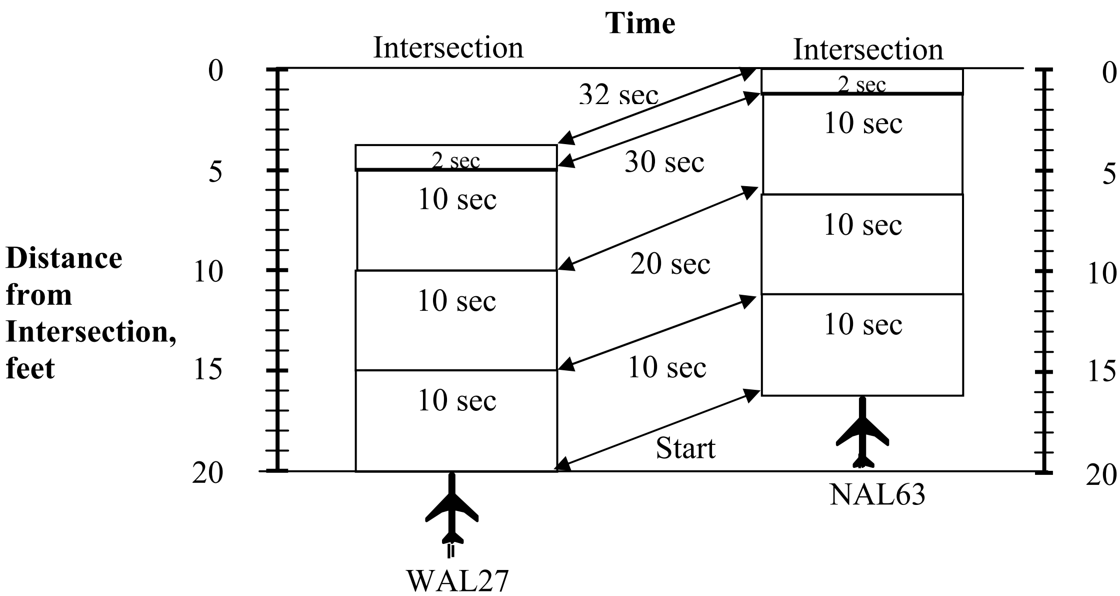
5. You filled in the table to find the answer. Can you think of a faster way to find the answer? If so, describe the faster way. **For Flight WAL27, multiply 2 seconds per foot by 20 feet to obtain 40 seconds. For Flight NAL 63, multiply 2 seconds per foot by 16 feet to obtain 32 seconds.**

6. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

Activity 2.3B—Stacking Blocks:

Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the 5-foot blocks added together. To get the answer in seconds, add the 10-second blocks.

As the blocks for Flight NAL63 are stacked, it will become clear that the answer is a little more than 30 seconds. In particular, Flight NAL63 has to go only 1 foot past 30 seconds. This foot corresponds to 2 seconds. So Flight NAL 63 will arrive in 32 seconds. At this time, Flight WAL27 is still 4 feet from the intersection. So a conflict will *not* occur.



1. How many seconds will it take each plane to travel to the point where the two routes meet?

WAL27 ___**40**___ seconds NAL63 ___**32**___ seconds

2. Will the two planes meet at the point where the two routes come together? ___**No**___

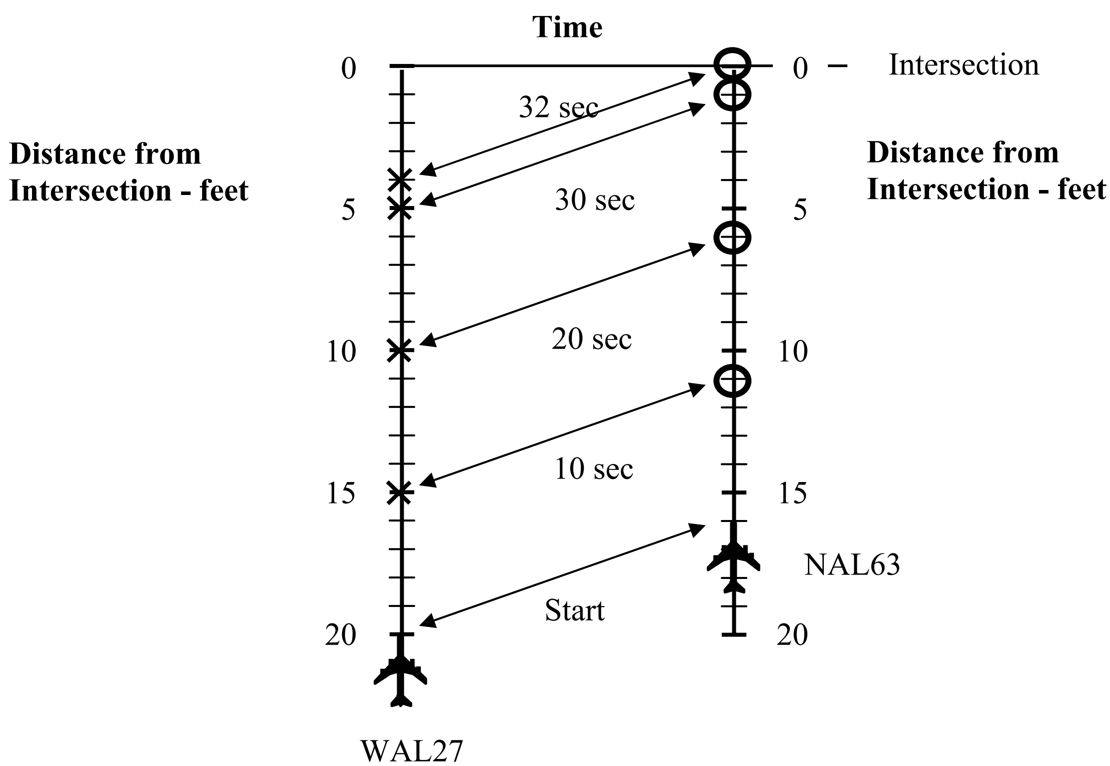
3. Why or why not? **At 32 seconds, when Flight NAL 63 arrives at the intersection, Flight WAL27 will be 4 feet from the intersection. So the planes will not meet.**

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

Activity 2.3C—Plot Points on Lines:

Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the position of each plane at 10-second intervals.

As the points for Flight NAL63 are plotted, it will become clear that the answer is a little more than 30 seconds. In particular, Flight NAL63 has to go only 1 foot past 30 seconds. This foot corresponds to 2 seconds. So Flight NAL 63 will arrive in 32 seconds. At this time, Flight WAL27 is still 4 feet from the intersection. So a conflict will *not* occur.



1. How many seconds will it take each plane to travel to the point where the two routes meet?

WAL27 ___**40**___ seconds NAL63 ___**32**___ seconds

2. Will the two planes meet at the point where the two routes come together? ___**No**___

3. Why or why not? **At 32 seconds, when Flight NAL 63 arrives at the intersection, Flight WAL27 will be 4 feet from the intersection. So the planes will not meet.**

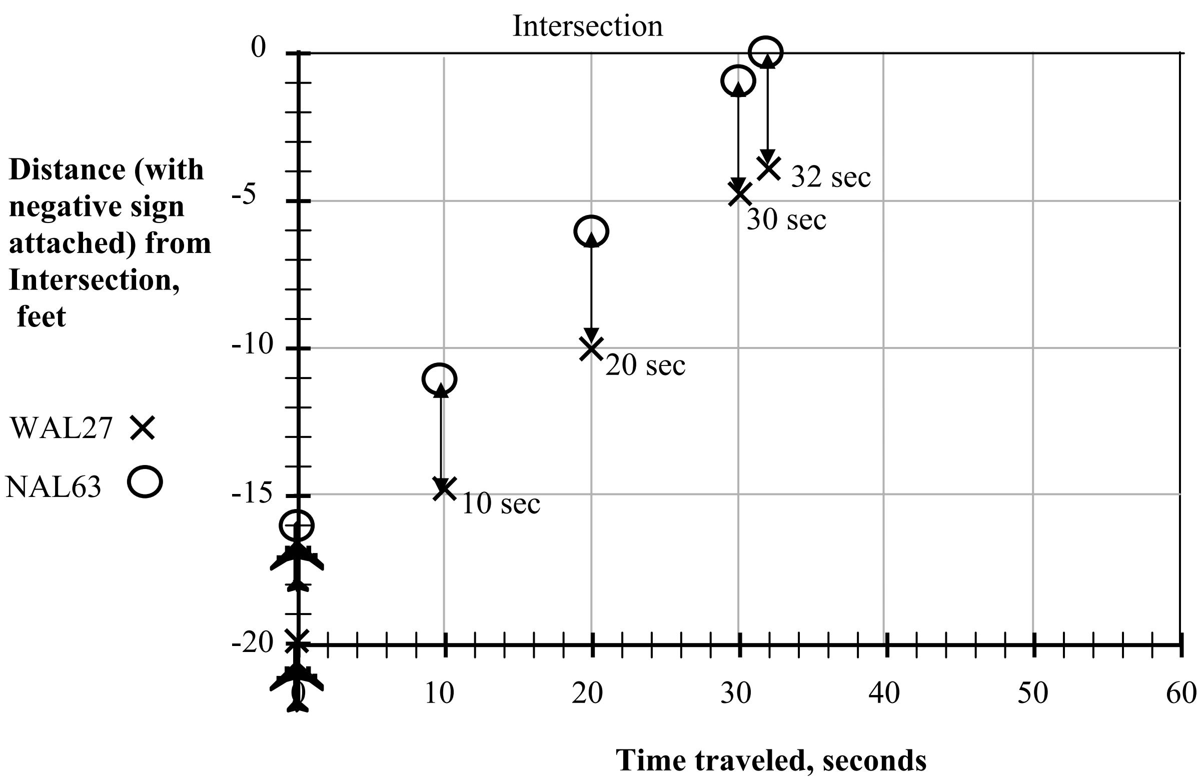
Activity 2.3C (cont.)

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

Activity 2.3D—Plot Points on a Grid:

Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following graph shows the position of each plane at 10-second intervals.

As the points for Flight NAL63 are plotted, it will become clear that the answer is a little more than 30 seconds. In particular, Flight NAL63 has to go only 1 foot past 30 seconds. This foot corresponds to 2 seconds. So Flight NAL 63 will arrive in 32 seconds. At this time, Flight WAL27 is still 4 feet from the intersection. So a conflict will *not* occur.



1. How many seconds will it take each plane to travel to the point where the two routes meet?

   WAL27 **__40__** seconds NAL63 **__32__** seconds

2. Will the two planes meet at the point where the two routes come together? **__No__**

3. Why or why not? **<u>At 32 seconds, when Flight NAL 63 arrives at the intersection, Flight WAL27 will be 4 feet from the intersection. So the planes will not meet.</u>**

Activity 2.3D (cont.)

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

Activity 2.3E—Derive the Distance-Rate-Time Formula:

In 4 seconds, each plane travels 0.5 feet/second x 4 seconds = 2.0 feet.

In 5 seconds, each plane travels 0.5 feet/second x 5 seconds = 2.5 feet.

In 6 seconds, each plane travels 0.5 feet/second x 5 seconds = 3.0 feet.

To find the distance each plane travels in 14 seconds, multiply 0.5 feet/second by 14.

To find the distance each plane travels in 20 seconds, multiply 0.5 feet/second by 20. The result is 10.0 feet.

1. In 4 seconds, each plane travels **0.5 feet/second** × **4 seconds** = **2.0** feet.
2. In 5 seconds, each plane travels **0.5 feet/second** × **5 seconds** = **2.5** feet.
3. In 6 seconds, each plane travels **0.5 feet/second** × **6 seconds** = **3.0** feet.

4. How could you use multiplication to find the distance each plane travels in 14 seconds? **Multiply 0.5 feet/second by 14 seconds.**

5. Use the formula to find the distance traveled by each plane in 20 seconds.
   In 20 seconds, each plane travels **10** feet.

Activity 2.3F—Use the Distance-Rate-Time Formula:

Calculate the time it takes each aircraft to travel to the intersection.

The travel times are:

WAL27: t = 20 feet / 0.5 feet per second = 40 seconds

NAL63: t = 16 feet / 0.5 feet per second = 32 seconds

Since the times are different, a conflict will not occur.

Flight WAL27 $t = \dfrac{\text{20 feet}}{\text{0.5 feet per second}}$ = **40** seconds

(Hint: Divide 20 by 0.5 .)

Flight NAL63 $\mathbf{t = \dfrac{\text{16 feet}}{\text{0.5 feet per second}}}$ = **32** seconds

1. How many seconds will it take each plane to travel to the point where the two routes meet?

   WAL27 **40** seconds NAL63 **32** seconds

2. Will the two planes meet at the point where the two routes come together? **No**

3. Why or why not? **At 32 seconds, when Flight NAL 63 arrives at the intersection, Flight WAL27 will be 4 feet from the intersection. So the planes will not meet.**

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

Activity 2.3G—Graph Two Linear Equations:

Flight NAL63 arrives at the intersection first, in 32 seconds.
Flight WAL27 arrives at the intersection 8 seconds later, at 40 seconds.
Since the times are different, a conflict will not occur.

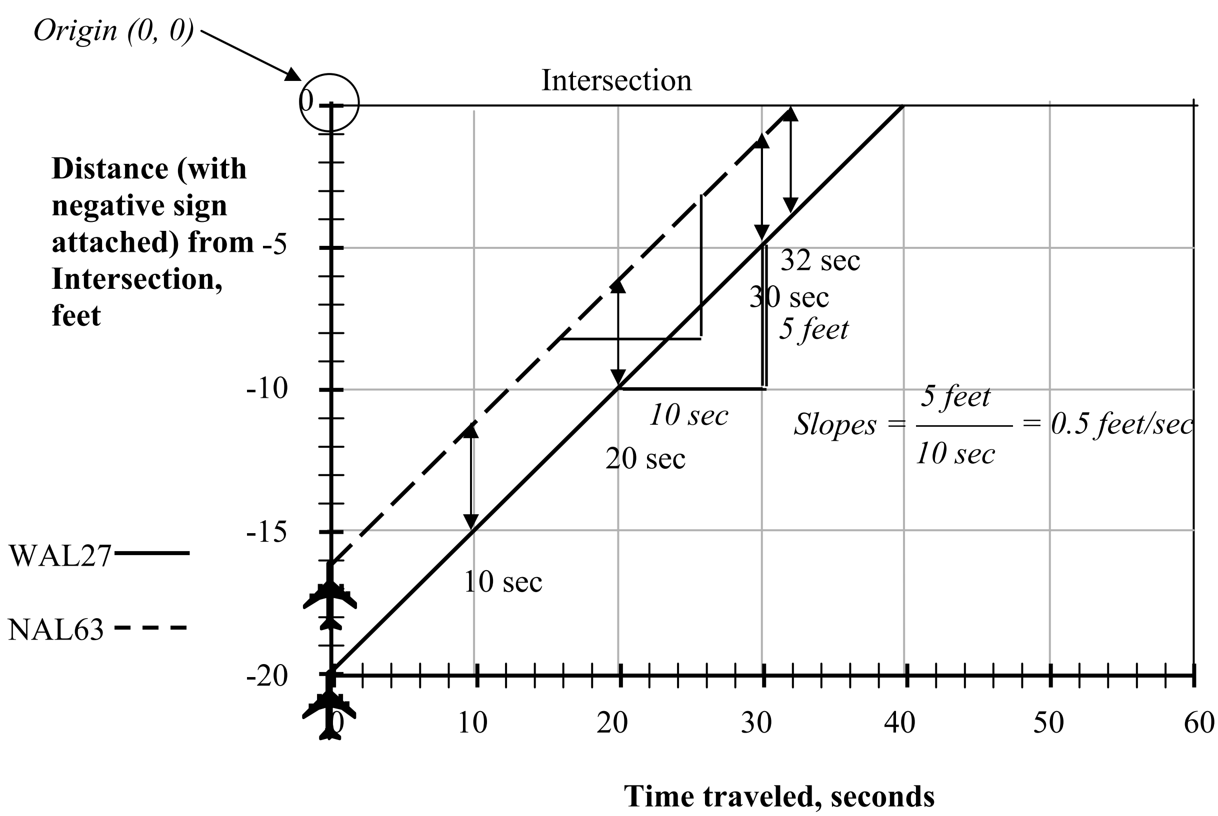
WAL27 $y = 0.5x - 20$

x	y
0	**-20**
10	**-15**
20	**-10**
30	**-5**

NAL63 $y = 0.5x - 16$

x	y
0	**-16**
10	**-11**
20	**-6**
30	**-1**

The slope of each line is 0.5 feet/sec, the speed of each plane.
Since the lines have the same slope, the lines are parallel.
The vertical intercept of each line (-20 and -16, respectively) corresponds to each plane's initial distance (20 feet and 16 feet, respectively) from the intersection.



Activity 2.3G (cont.)

1. How many seconds will it take each plane to travel to the point where the two routes meet?

   WAL27 **40** seconds NAL63 **32** seconds

2. Will the two planes meet at the point where the two routes come together? **No**

3. Why or why not? **At 32 seconds, when Flight NAL 63 arrives at the intersection, Flight WAL27 will be 4 feet from the intersection. So the planes will not meet.**

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

5. Write the number that is the slope of the solid line representing Flight WAL27. **0.5 feet/second**

6. Write the number that is the slope of the dotted line representing Flight NAL63. **0.5 feet/second**

7. What information does the slope of each line tell you about each plane?

   The slope of each line is 0.5 feet/second, the speed of the plane.

Activity 2.4—After the Experiment and Activity 2.5—Posttest:

The plane speeds are the same.
The planes are different distances from the finish point.

General Problem:

Suppose two planes are traveling at the same constant (fixed) speed on two different routes and the planes are different distances from the point where the two routes come together.

Since the planes are traveling at the same constant (fixed) speed and must each travel a different distance to the point of intersection, the planes will arrive at the intersection at different times. So the planes will not meet at the point where the routes come together.

Also, since the planes are traveling at the same constant (fixed) speed, their separation remains the same. So at the intersection, the separation between the planes will be the same as their initial separation.